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Parametrization of cosets for AdS_5xS^5 superstring action

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ABSTRACT

A formulation recently proposed [arXiv:1506.07706] as an alternative to the usual coset PSU(2,2|4)/USp(2,2)USp(4) for the superspace geometry of the Type IIB superstring on an $AdS_5 \times S^5$ background is shown to be a particular parametrization of this coset. Standard methods can then be applied.

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Various parametrizations have appeared in the literature for the coset space PSU(2,2|4)/USp(2,2)USp(4) found to describe the Type IIB superstring on the background $\mathrm{AdS}_5 \times \mathrm{S}^5$ [1]. The original exponential parametrization was not the most convenient; algebraic ones were found to have some advantages [2,3]. In either case some properties can be improved by choice of ordering of various components by factorization.

One such factorization is [2]

$$g = \begin{pmatrix} 1 & \theta \\ \bar{\theta} & 1 \end{pmatrix} \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$$

where the global PSU(2,2|4) acts on the left and the local USp(2,2)USp(4) on the right. This has two advantages: (1) x acts directly as an element of the coset SU(2,2)/USp(2,2) (AdS_5) and y as SU(4)/USp(4) (S^5). Thus the fermions θ are invariant under the local groups, while the bosonic local-group invariants are (in terms of the antisymmetric metric C of USp(2,2) and USp(4))

$$X = xCx^T \ , \quad Y = yCy^T$$

(2) Global PSU(2,2|4) transformations act on $(1, \bar{\theta})x$ and $(\theta, 1)y$, and therefore projectively on all the fermions (not mixing with the bosons):

$$g' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} g \quad \Rightarrow \quad \theta' = (a\theta + b)(d + c\theta)^{-1} \ , \quad \bar{\theta}' = (d\bar{\theta} + c)(a + b\bar{\theta})^{-1}$$

The action can then be constructed as quadratic in the globally invariant currents $J = g^{-1}dg$ (including the Wess-Zumino term [4]). The local invariance of the action means x and y automatically combine into X and Y.

However, the above parametrization has unusual hermiticity properties (unless Wick rotated to $\mathrm{PSL}(4|4)/\mathrm{Sp}(4)^2$). To satisfy

$$gMg^\dagger = M \ , \quad M = \begin{pmatrix} \Upsilon & 0 \\ 0 & I \end{pmatrix}$$

(where Υ is the indefinite hermitian metric of SU(2,2)), we make a simple modification, a redefinition of x and y, not affecting the PSU(2,2|4) transformations of the fermions nor the local transformations of anything:

$$g = \begin{pmatrix} 1 & \theta \\ \bar{\theta} & 1 \end{pmatrix} \begin{pmatrix} (1 - \theta\bar{\theta})^{-1/2} & 0 \\ 0 & (1 - \bar{\theta}\theta)^{-1/2} \end{pmatrix} \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$$

Then

$$\theta^\dagger = -\bar{\theta}\Upsilon \quad (\bar{\theta}^\dagger = -\Upsilon\theta) \ , \quad x\Upsilon x^\dagger = \Upsilon \ , \quad yy^\dagger = I$$

The results of [5] then follow straightforwardly from the previous coset methods.

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